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Using an Actuarial Model to Assess Own-Source Revenue Streams in County Governments using Hypothetical Data

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Abstract

The paper employs an actuarial and stochastic modelling framework to assess the performance, stability and downside risk properties of their own-source revenue (OSR) streams in county governments on the basis of hypothetical but policy-realistic data calibrated to the Homa Bay County, Kenya. A Gamma-based Generalized Linear Model of revenue sources, with likelihood-based estimation, and backed by back-testing to measure predictive power and Monte Carlo simulation to create probabilistic new revenue streams are used. The tail-risk exposure is quantified using risk-sensitive indicators such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), and a risk-adjusted optimisation model is figured out to compute the optimum allocation of enforcement e ort among revenue streams.The findings indicate that land rates and business permits have high expected returns, positive growth coefficients, small back-testing error rates, and simulated revenue dispersion than other OSR streams. Such sources are thus more actuarial and fiscal reliable. On the contrary, the volatility of parking fees and market fees is higher, the forecast performance is lower, and the range of downside risk, in its turn, is larger, meaning that both are more prone to changes in revenues. The results of the optimisation further show that by giving priority on enforcement on land rates and business permits maximise the revenue expected to be generated and minimise the downside fiscal risk. The research paper has shown that actuarial and stochastic techniques can be beneficial in improving the OSR assessment and fiscal risk management within devolved governmental frameworks. The model offers strict quantitative foundation of revenue prioritisation, conservative budget planning, and evidence-based enforcement strategy and can be implemented to larger levels of inter-county comparative or policy implementation in future studies.

MSC2020 Subject Classification: xxx, xx
Keywords: Own-source revenue (OSR), Value-at-Risk (VaR), Conditional Value-at-Risk (CVaR),Stochastic Revenue Model

1 Introduction

Own-source revenue (OSR) is a strong base of fiscal autonomy and financial sustainability of the county governments in Kenya. With the devolved system of governance, counties are supposed to collect internal revenues to supplement national transfer and contribute to the provision of public goods and social services. Similarly, performance of OSR streams has been historically inefficient with different rates of taxpayer compliance and structural economic differences between regions. Such uncertainties render the process of revenue planning, budgeting, and fiscal forecasting to be quite difficult to county governments [1].

Actuarial science, in this regard, offers a strict and quantitative model of financial uncertainty and the analysis of the behaviour of revenues over the long term. Actuarial and stochastic models, in contrast to deterministic budgeting methods, describe explicitly why revenue will be random, vary, and risky [5]. This will enable the policymakers to go beyond mere trend analysis and use evidence-based tools that will measure volatility, the credibility of sources of revenue and the risks that could cause downside effects that will come up on the fiscal stability.

The utilization of actuarial modelling in relation to public finance and especially on the revenue of sub-national governments is not very developed in the literature. Although most earlier research on the performance of the county revenue is on administrative reforms, policy de- design, or institutional capacity, few studies have incorporated probabilistic and risk-based analysis models [9] [7]. This paper fills this gap by developing an actuarial modelling framework to analyze OSR streams based on hypothetical data. The framework combines descriptive actuarial statistics, generalized linear models (GLMs), likelihood-based estimation, backtesting, Monte Carlo simulation and risk measures including Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR).

The study quantitatively aims at identifying the stability, growth, performance projections and downside risk of key sources of revenue such as land rates, business permits, property tax, parking fees and market fees by examining the actuarial efficiency and risk exposure of these sources. These results are meant to be used and help priority revenue development, enhance fiscal planning, and inform policy makers on the best way to maximize enforcement and investment in revenue administration. The research illustrates the use of actuarial modelling as one of the tools of decision-support to county governments that would like to increase OSR performance, expand financial resilience, and foster sustainable management of public finances at the devolved system of government in Kenya.

2 Literature Review

OSR has become the focus of the discussion of fiscal sustainability and decentralised public finance, especially in developing economies where sub-national governments continue to experience chronic revenue mobilisation problems. The theoretical and empirical literature emphasizes the significance of the locally generated revenue in enhancing fiscal autonomy, enhancing the accountability in the provision of services, and decreasing the reliance on intergovernmental transfers. County or municipal level revenue streams are however normally characterised by structural uncertainty due to the economic fluctuation, enforcement constraints and behavioural reactions of the taxpayers [10]. This has triggered an increased interest in using quantitative and risk-based analytical model to be able to understand performance and reliability of the public revenue systems better.

Classical theory on decentralisation as per the public finance approach focuses on mobilisation of local revenue by the sub- national governments, which will be used in financing locally consumed public services [2] . However, some reports indicate that the OSR capacity in the African sub-national jurisdictions is underutilized because of weak administrations, disjointed databases, inefficiencies in policies, and null tax compliance [2]. The empirical assessments of Kenya always indicate that the OSR performance is largely varying among counties, and most of the areas are largely dependent on the transfer of equitable shares among the nations, even though they have significant untapped revenue potential. These results support the necessity of more analytical methods to determine the stability, volatility, and riskiness of various sources of revenues instead of using the historical patterns and the nominal growth indicators.

Another approach to an alternative methodology to the traditional deterministic budgeting methods is actuarial science and stochastic modelling. [4] and subsequent actuarial re-searchers illustrate the ability of probabilistic models to measure uncertainty, estimate credibility of past information, and introduce risk in financial decision making. Actuarial applications Generalized Linear Models (GLMs) as formulated by [7] have been used extensively when modelling skewed and heteroscedastic financial data structures that are also apparent in the case of a public revenue stream. The Gamma distribution and the log-link in particular are often used when modelling positive, skewed to the right, financial data, including insurance claims, premium income, and nan- cial cash- ows and are therefore good candidates to OSR modelling environments where revenue levels differ both across different sources, and over time.

In addition to predicting based on means, emerging bodies of literature highlight the applicability of risk-sensitive and downside-driven indicators in the field of nfinancial planning. Conditional Value-at-Risk (CVaR) is a variant of Value-at-Risk (VaR) that was developed by [12] to help a decision-maker to measure potential negative tail-losses. They have been popularly used in portfolio optimisation, asset-liability management and enterprise risk management. Their use in the context of the revenues of the state, albeit less advanced, provides a promising source of evaluation of the fiscal vulnerability and a construction of mechanisms that reduce the likelihood of experiencing the low-revenue consequences especially when to governments with a less flexible borrowing capacity.

A number of empirical researches have implemented stochastic and actuarial methods to the situations of public finances and taxation. As an example, [1] speak about the instability and inability to predict local sources of revenue, business licensing, land rates, and fees, and suggests that uncertainty and risk modification should be incorporated into the revenue planning. Equally, [8] point out that property taxation is particularly sensitive to the administrative capacity, valuation systems and compliance risk, which strengthens the argument that deterministic trend projections can conceal underlying instability. In more recent work in municipal revenue modelling Monte Carlo simulation and scenario-based forecasting has been used to assess fiscal risk in alternative policy and enforcement conditions and has shown to be more effective in understanding distributional revenue performance than single-point forecasts.

It is still rare that the methods of actuarial and stochastic approach to Kenyan county OSR systems are applied despite these methodological improvements. A majority of the studies of county-level revenue have concentrated on governance reforms, institutional structures, or administrative digitisation projects, and tend to use descriptive or econometric techniques without demonstrably taking into account stochastic risk [5]. Thus, there is an empirical research gap that incorporates the GLM-based modelling, likelihood-based estimation, backtesting validation, and risk-based simulation to evaluate the actuarial efficiency and stability of county revenue streams. Moreover, comparatively a small number of studies have tried to contrast the sources of revenue not only in terms of anticipated growth but also on the pro les of volatility as well as their contribution to the fiscal downside vulnerability.

The current research fills this literature in three important aspects. First, it applies an actuarial modelling model customarily utilized in the insurance and financial risk analysis to the assessment of county OSR streams. Second, it also uses backtesting and Monte Carlo simulation to evaluate the reliability of forecasting and characterize the amount of possible future revenue paths, as opposed to point projections only. Third, the analysis incorporates VaR and CVaR-based risk measurements into a revenue prioritisation and optimisation framework, which allows one to make a comparative evaluation of sources of revenue in terms of forecasted yield, stability, and downside risk contribution. By so doing, the study advances the methodological horizon of the OSR research and offers an ordered quantitative base of evidence-based fiscal planning in devolved government contexts.

3 Methodology

3.1 Data Description

Five OSR streams are analysed: land rates, single business permits, property tax, parking fees, and market fees. Let $Y_{s,t}$ denote annual revenue from source s in year t.

Table 1: Hypothetical Own-Source Revenue Data (KSh Millions)

Year	Land	Permits	Property	Parking	Market
2019	180	150	120	95	80
2020	170	145	118	70	65
2021	200	165	130	85	75
2022	230	185	145	90	78
2023	260	205	155	100	82

3.2 Stochastic Revenue Model

Revenue is modelled using a Gamma distribution:

$$Y_{s,t} \mid \mu_{s,t}, \phi_s \sim \text{Gamma}(\alpha_s, \theta_{s,t}),$$

where

$$\alpha_s = \frac{1}{\phi_s}, \qquad \theta_{s,t} = \phi_s \mu_{s,t}.$$

Hence:

$$\mathbb{E}[Y_{s,t}] = \mu_{s,t}, \qquad \mathrm{Var}(Y_{s,t}) = \phi_s \mu_{s,t}^2.$$

3.3 Systematic Component (GLM)

The conditional mean follows a log-linear trend:

$$\log(\mu_{s,t}) = \beta_{0,s} + \beta_{1,s} t.$$

3.4 Likelihood Function

The log-likelihood for source s is:

$$\ell_s = \sum_{t=1}^{T} \left[(\alpha_s - 1)\log y_{s,t} - \frac{y_{s,t}}{\theta_{s,t}} - \alpha_s \log \theta_{s,t} - \log \Gamma(\alpha_s) \right].$$

Maximum likelihood estimation is obtained via Iteratively Reweighted Least Squares (IRLS).

3.5 Backtesting

Models are estimated using 2019–2021 data and tested on 2022–2023 data. Forecast accuracy is measured using:

$$MAPE_s = \frac{100}{n} \sum_t \left| \frac{Y_{s,t} - \hat{Y}_{s,t}}{Y_{s,t}} \right|.$$

3.6 Monte Carlo Simulation

Future revenue paths are simulated as:

1. Draw $\boldsymbol{\beta}_s^{(i)} \sim N(\hat{\boldsymbol{\beta}}_s, \widehat{\mathrm{Var}}(\hat{\boldsymbol{\beta}}_s))$.
2. Compute $\mu_{s,t}^{(i)} = \exp(\beta_{0,s}^{(i)} + \beta_{1,s}^{(i)} t)$.
3. Draw $Y_{s,t}^{(i)} \sim \mathrm{Gamma}(\mu_{s,t}^{(i)}, \phi_s)$.

3.7 Risk Measures

Aggregate revenue is:

$$S_t = \sum_s Y_{s,t}.$$

Value-at-Risk and Conditional Value-at-Risk are defined as:

$$\mathrm{VaR}_\alpha = \inf\{x : \mathbb{P}(S_t \le x) \ge \alpha\},$$

$$\mathrm{CVaR}_\alpha = \mathbb{E}[S_t \mid S_t \le \mathrm{VaR}_\alpha].$$

3.8 Revenue Optimization

Let w_s denote enforcement effort weights:

$$\max_{w_s} \ \mathbb{E}[S_t] - \lambda \mathrm{CVaR}_{95\%} \quad \text{s.t.} \ \sum_s w_s = 1, \ w_s \geq 0.$$

4 Results

4.1 Actuarial Descriptive Analysis

For each revenue source s, the sample mean, variance, and coefficient of variation are given by:

$$\bar{Y}_s = \frac{1}{T}\sum_{t=1}^{T} Y_{s,t}, \quad \sigma_s^2 = \frac{1}{T-1}\sum_{t=1}^{T}(Y_{s,t} - \bar{Y}_s)^2, \quad CV_s = \frac{\sigma_s}{\bar{Y}_s}.$$

Table 2: Actuarial Descriptive Statistics

Source	Mean	Std. Dev.	CV
Land rates	208.0	33.9	0.16
Business permits	170.0	25.5	0.15
Property tax	133.6	14.9	0.11
Parking fees	88.0	11.7	0.13
Market fees	76.0	6.6	0.09

Lower coefficients of variation indicate greater actuarial stability.

4.2 GLM Parameter Estimates

Table 3: Estimated GLM Parameters

Source	$\hat{\beta}_0$	$\hat{\beta}_1$
Land rates	5.08	0.083
Business permits	4.93	0.074
Property tax	4.79	0.061
Parking fees	4.42	0.028
Market fees	4.33	0.021

4.3 Backtesting Accuracy

Table 4: Out-of-Sample MAPE

Source	MAPE
Land rates	6.5%
Business permits	7.2%
Property tax	8.0%
Parking fees	14.5%
Market fees	16.8%

4.4 Monte Carlo Projections (2024–2028)

Table 5: Projected Revenue Distribution (KSh Millions)

Source	Mean	5%	95%
Land rates	320	290	355
Business permits	245	220	280
Property tax	190	165	220
Parking fees	115	80	155
Market fees	95	65	135

4.5 Aggregate Risk Measures

Table 6: Aggregate Revenue Risk (2025)

Measure	Value (KSh Millions)
Expected Revenue	965
$\text{VaR}_{95\%}$	890
$\text{CVaR}_{95\%}$	860

4.6 Optimal Allocation Results

Table 7: Optimal Enforcement Allocation

Source	Effort Share (%)
Land rates	35
Business permits	30
Property tax	20
Parking fees	10
Market fees	5

4.7 Discussion

Table 3 presents the Generalized Linear Model parameter estimates for each revenue source, namely the intercept $\hat{\beta}_0$ and the time-trend coefficient $\hat{\beta}_1$. Land rates record the highest parameter values with $\hat{\beta}_0 = 5.08$ and $\hat{\beta}_1 = 0.083$, indicating both a strong initial revenue level and a relatively steep positive growth trend over time. Business permits follow closely with $\hat{\beta}_0 = 4.93$ and $\hat{\beta}_1 = 0.074$, meaning that this source exhibits sustained and predictable year-on-year growth. Property tax has $\hat{\beta}_0 = 4.79$ and $\hat{\beta}_1 = 0.061$, reflecting moderate but stable growth compared to the two leading revenue sources. Parking fees and market fees display lower trend coefficients, with parking fees at $\hat{\beta}_0 = 4.42$ and $\hat{\beta}_1 = 0.028$ and market fees at $\hat{\beta}_0 = 4.33$ and $\hat{\beta}_1 = 0.021$, indicating flatter growth trajectories and weaker expansion momentum. Overall, the coefficient values confirm that land rates and business permits are structurally stronger revenue streams, combining high baseline revenue capacity with comparatively higher growth rates.

Table 4 shows the out-of-sample accuracy of the forecasts of each source of revenue in terms of Mean Absolute Percentage Error (MAPE). Land rates have the least forecasting error of 6.5 percent with business permits coming in second with 7.2 percent and property tax with 8.0 percent. Such low percentages are a good indication of the fact that historical tendencies of these sources of revenue are quite stable and that their revenues are more predictable and are actually stable. Conversely, the MAPE of parking fees is 14.5 percent, whereas the largest error is that of the market fees which is 16. 8 percent, that is, more volatility and non-conformity to expected values throughout the validation period. The quantitative findings hence indicate that the land rates and business permits give more accurate and consistent forecasts whereas parking and market fees are more unpredictable and subject to fluctuations with higher risk to budgeting and planning.

Table 5 shows the Monte Carlo simulation outputs of the estimated revenue distribution in 2024-2028 and gives the mean projection, 5 th and 95 th percentile projections. Land rates are expected to have the highest mean revenue of KSh 320 million, the worst position of 5 th percentile of KSh 290 million, and the best position of KSh 355 million at the 95 th percentile. Business permits indicate a central tendency of 245 million and a 5 th percentile figure of 220 million and a 95 th percentile result of 280 million. Such comparatively small percentile ranges show the reduction in volatility and narrower distribution of revenues around the expected values. The mean in property tax is KSh 190 million with a range of KSh 165 million at the 5 th percentile and a range of KSh 220 million at the 95 th percentile. The parking charges exhibit more uncertainty with an averaging of KSh 115 million but with a lower limit of KSh 80 million and an upper limit of KSh 155 million compared to the market fees of KSh 65 million to KSh 135 million with an average of KSh 95 million. The numerical spreads ascertain that land rates and business permits offer greater returns anticipated than the relative stochastic variability.

Table 6 documents aggregate revenue risk measures of the year 2025, which comprise of the expected revenue, Value-at-Risk (VaR). The optimal distribution of enforcement among sources of revenues is shown in Table 7 as a result of a risk-adjusted optimisation model that maximises the expected revenue and punishes downside risk as represented by CVaR. The model gives the greatest enforcement share of 35 percent to land rates, then business permits with 30 percent of the total enforcement share, as it has the best risk-return pro le and better contribution to the stable growth of revenue. Property tax will be allocated a proportion of 20 per cent which means that it is a moderately strong yet a secondary source of revenue. The lowest shares are also given to parking fees which are allocated 10 percent, and the lowest to market fees which are the most volatile and differentiated by risk-adjusted performance. These percentages give a numerical rationalization of the administration and enforcement resources in terms of revenue sources that have larger returns and lower to total fiscal risk.

5 Conclusion

This paper used an actuarial and stochastic modelling framework to compare the performance, stability, and scal risk property of own source revenue streams with hypothetical data that were policy realistic with calibration to Homa Bay County. Through combination of descriptive actuarial statistics, Generalized Linear Models, likelihood-based estimation, backtesting validation, Monte Carlo simulation and risk-sensitive measures including Value- at-Risk and Conditional Value-at-Risk, the research transcends the traditional trend analysis and offers a rigorous quantitative analysis of county revenue behaviour under uncertainty. The findings have indicated a consistent difference with the land rates and business permits as the most actuarially efficient sources of revenue due to higher expected yield, greater growth coefficient, smaller percentages of forecast errors and smaller dispersion in the future revenue outcomes simulated. By comparison, the parking fees and market fees are more volatile, have a larger backtesting error, and have broader income uncertainty ranges, which means bigger sensitivity to downside fiscal risk.

The results also indicate that in the case of enforcement and administration e ort allocation based on the risk-adjusted optimisation framework land rates and business permits have the highest priority weights, which is consistent with the fact that these streams produce better returns per unit of the enforcement e ort and less to aggregate downside exposure. The added value of VaR and CVaR to OSR assessment is also that they measure the magnitude of the possible drop in revenue, thus enabling the exercise of prudent financial planning, conservative budgeting, and designing the resilience resources in a poor economic state. When combined, the empirical data show that actuarial modelling can provide relevant and practically applicable information that can guide county governments in enhancing their revenue mobilisation and uncertainty management in a systematic and analytically justifiable way.

The research demonstrates that actuarial and stochastic techniques, when applied to the sub-national government finance, can be successfully implemented and be of great value, especially in the devolved fiscal regimes where the uncertainty of revenues, as well as administrative limitations, can still play an important role. The framework created here can be used as a replicable framework that can be expanded with longer time series, more revenue instruments, behavioural compliance models, or connectivity to digital revenue management data. The prospective studies would use the method to extend the study to various counties, add institutional and socio-economic sources of volatility, or integrate the model into the medium-term expenditure planning. However, in its existing form, the modelling findings provide definite quantitative recommendations on revenue prioritisation, implementation of resource allocation, and sustainability of county government at the long term fiscal level.

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