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Mathematical modeling of burglary dynamics incorporating unemployment in Kenya

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ABSTRACT

Burglary remains a critical socioeconomic challenge in Kenya, with unemployment identified as a key driver. This study presents a deterministic compartmental model using ordinary differential equations to explore how unemployment influences burglary dynamics in Kenya. The population is divided into five classes: susceptible, exposed, active burglars, incarcerated individuals, and those released from prison. Analytical results demonstrate that the model is well-posed, with positive and bounded solutions. Using the next-generation matrix method, we compute the effective reproduction number R_e *to analyze the stability of the burglary-free equilibrium point which is locally and globally asymptomatically stable when* $R_e < 1$ *and the burglary-endemic equilibrium point which is both locally and globally asymptotically stable when* $R_e > 1$*, indicating persistent burglary. The sensitivity analysis reveals that the employment rate* ω *is the most critical parameter for control, with a higher employment rate that significantly reduces* R_e*. Numerical simulations using ODE-45 in MATLAB software corroborate these findings, showing that increased job opportunities drastically lower the incidence of burglary. This study provides insights for policy formulation, highlighting employment creation as an effective strategy to reduce burglary. This work contributes to social mathematical modeling by bridging socioeconomic factors with crime dynamics, offering a replicable framework for similar contexts. The model introduces novel elements such as recidivism and unemployment-linked transmission dynamics, offering a unique contribution to crime modeling in Kenya.*

Keywords: Mathematical model, Unemployment, effective reproduction number

1 Introduction

Crime is an unlawful act constituting an offense punishable by law, or more generally, an intentional act that is socially harmful and prohibited under legal statutes according to Mataru [8] and Musoi[14]. According to Siegel [19], crime is a violation of societal rules of behavior expressed and interpreted by a criminal legal code created by people holding economic, social and political power.

Crimes are classified into four main categories; felony, misdemeanor, minor offenses, and capital offenses [1]. Crimes such as arson, manslaughter, kidnapping, abduction, cybercrime, drug possession, sex crimes, rape, assault, affray, riot, forgery, creating disturbances, offensive conduct, malicious damage, and burglary are categorized as felonies. Felonies are crimes punishable without the proof of a previous conviction by death or imprisonment. Petty theft, simple assault, trespass, and careless driving are categorized as misdemeanors which are lesser crimes punishable by a fine or up to three years of imprisonment. Petty offenses are less severe and punishable by a small fine, a warning, community service, or less than a year of imprisonment. Examples include obtaining property by false pretense, creating a public nuisance, loitering with intent to commit prostitution, public intoxication, being drunk and disorderly, hawking without a permit, and littering. Capital offenses are the most serious and punishable by death, including murder, homicide, treason, robbery with violence, and espionage.

Burglary involves unlawful entry into a building, home, or premises with the intent to steal. According to the Kenyan Penal Code (2009), Chapter 63, Section 304(2), burglary is defined as breaking into a building, tent, or any other premises with the intent to commit a felony or having committed a felony in such a premises and breaking out thereof at night. The seriousness of burglary is reflected in its punishment, which can include imprisonment for up to seven years.

The desire of a country to provide maximum protection to its citizens is reflected in the severity of the punishments imposed for violating security and privacy. Burglary is a common property crime that occurs worldwide. In recent decades, it has become a major problem in Kenya, with daily cases recorded showing an increasing trend in rural and urban areas according to Mburu & Bakillah [10]. This reflects a high national incidence of burglary, imposing enormous financial and social problems on victims. Insurance companies often do not provide comprehensive coverage for homeowners, and increased premiums have accelerated burglary rates. Beyond financial loss, burglary victims experience emotional and psychological stress. Discovering that your home has been ransacked and your valuables have disappeared creates feelings of outrage, frustration, and vulnerability, with long-lasting effects on mental health.

Burglary spreads more rapidly like infectious disease in urban areas of Kenya than in rural areas due to the nature of settlements. Burglars exploit urban congestion and complex corridors, making it difficult for police to track and arrest them[9]. Burglars use four primary methods to gain access to premises, that is, threats, which is the use of the command word to gain entry, artifice is the use of tricks, collusion is the agreement with an insider, and aperture is exploiting openings in buildings[1]. Burglary continues to threaten peace and security in Kenya.

According to the National Crime Research Center (2023), professional burglars who rely on burglary as a livelihood have increased due to economic stress. In Kenya, more than one million graduates enter the job market annually, while demand remains stagnant or declining[17]. Unemployment and low wages drive property crimes, such as burglary, as individuals can resort to illegal means to meet basic needs, especially when the perceived gain of the crime is greater than the potential punishment[2]. The interaction between employed criminals and unemployed people further exacerbates the problem, as the unemployed can be influenced to commit burglary.

By reviewing the relevant literature, the study aims to establish the relationship between unemployment and crime, highlighting key factors that contribute to this social challenge. The research model developed by Mataru et al. [8] employs a compartmental approach to analyze unemployment-driven criminality, dividing the population N(t) into five groups that are unemployed U(t), susceptible to criminal influence S(t), active criminals C(t), vocational trainees V(t), and employed E(t). The model incorporates key parameters including the recruitment rate τ into unemployment, natural mortality rate μ, and transition rates between compartments. Individuals progress from unemployment to criminal exposure at a rate ϕ, while engagement in criminal activities occurs at rate β for the unemployed and $\beta(1\text{-}\theta)$ for the exposed population. Additional parameters include π for criminal recruitment from unemployment, φ for criminality-related mortality, ω for general vocational training enrollment, σ for unemployed individuals entering training programs, ρ for unemployed individuals transitioning to self-employment after skill acquisition, and ε for criminals reforming through rehabilitation programs.

Although Mataru et al [8] establish unemployment as a driver of crime in developing countries using compartmental models, their approach examines crime in general terms. Our study advances this work by specifically analyzing burglary patterns in Kenya, distinguishing it from other crime categories including white collar crimes such as tax fraud and evasion, violent crimes such as homicide, and other property crimes, including arson. Unlike previous generalized approaches, our model provides targeted insights through three key innovations: its exclusive focus on burglary dynamics, the introduction of a recidivism component via a released class to capture post-incarceration relapse behavior, and the explicit incorporation of employment rate ω into the force of influence β to model the unemployment burglary feedback loop.

Furthermore, recent advances in mathematical criminology have employed fractional-order models to account for memory effects on social behavior. For example, Soemarsono [20] and Sundar[21] used fractional differential equations to capture long-term influences in unemployment-driven crime. While our model remains within the classical ODE framework.

2 Model Formulation and Analysis

2.1 Model description and formulation

This study categorizes the total population $N(t)$ is divided into five compartments namely; Susceptible class $S(t)$, exposed class $E(t)$, burglary class $B(t)$, incarcerated class $I(t)$ and released class $R(t)$ are described as follows; The Susceptible class $S(t)$ are individuals in society who are not currently burglars but are at risk of being influenced to engage in burglary. The exposed class $E(t)$ is individuals who have been influenced through social interaction with active burglars but are not yet actively participating in burglary activities. The burglary class $B(t)$ consists of individuals actively engaged in burglary. They influence others in susceptible or exposed classes through their interactions, encouraging new entrants into criminal activity. The incarcerated class $I(t)$ consists of individuals who have been apprehended and detained by law enforcement as a result of their involvement in burglary activities. The released class $R(t)$ consists of individuals who have completed their jail term from the justice system.

Susceptible individuals are recruited at a constant rate Λ, and upon interaction with burglars, they are exposed to burglary activities at the rate β, while exposed individuals join the burglary class at the rate α. Burglary individuals who are proven guilty of committing such a crime are incarcerated at the rate γ, while some of the criminals die as a result of burglary-associated activities at the rate ρ . Individuals convicted for committing burglary are released at the rate ϕ. Upon release, individuals can either move to an exposed compartment at the rate $(1-\theta)$ or re-engage in burglary activities at the rate θ and μ representing the natural mortality rate. At a given time t the total population is given by ;

$$N(t) = S(t) + E(t) + B(t) + I(t) + R(t)$$

and β, which is the force of influence given by

$$\beta = \frac{\delta\tau(1-\omega)(B(t)+\lambda E(t))}{N(t)} \tag{1}$$

Where δ is the probability at which susceptible individuals become burglars upon contact with burglars, τ is contact rate between susceptible and burglary individuals, ω is the parameter accounting for employment rate which ranges from $(0 < \omega < 1)$ while $(1-\omega)$ is the rate of unemployed and λ is a modification parameter which accounts for the fact that exposed individuals to burglary have a higher probability of influencing others since they freely mingle with people in the population whereas the burglars live a secretive lives and fear being arrested thus minimal interaction with susceptible population.

2.2 Model Assumptions

The following assumptions are used to develop the model;

i. The population under the study is homogeneous

ii. Incarcerated individuals while in prison undergo temporal reform, therefore once released from jail they are initially not active in burglary; therefore some re-exposed and after sometime some resume to burglary practices due to economic hardship or lack of employment opportunities.

iii. Susceptible individuals cannot commit burglary without exposure.

iv. It is assumed that released individuals cannot return to a naive state of susceptibility. Once someone has been through the criminal justice system, their experience can make them less innocent or unaware of burglary compared to individuals in the $S(t)$ class.

v. An individual once exposed and becomes a burglar will remain a criminal until incarcerated, exit through natural death, or die as a result of burglary- associated activities.

vi. There is a constant recruitment into the study population.

vii. Exposed individuals are not easily recognizable in the population unless they commit a felony;

The summary of the model parameters are shown in Table 1.

Table 1. Model descriptions

Parameters	Model parameters descriptions
Λ	Natural recruitment rate to susceptible class
β	Rate at which susceptible individuals are exposed to burglary due to interaction
α	The rate at which exposed individuals proceed to burglary class
ρ	Mortality rate of burglars as a result of burglary associated activities
γ	Rate at which burglary individuals are being imprisoned
ϕ	Rate at which incarcerated individuals are released from prison
θ	The parameter captures recidivism. Is the rate at which criminals resume back to burglary activity after released from prison.
$1-\theta$	Rate at which some released individuals relapses back to exposed class
δ	The probability at which susceptible individuals become a burglar upon contact with burglars
τ	rate of contact between susceptible and burglary individuals
ω	Parameter accounting for employment rate
λ	Modification parameter accounting for the relative influence of the exposed individuals compared to burglary individuals
μ	Natural mortality rate

2.3 Burglary dynamics model flow diagram

Figure 1 is the schematic diagram representation of the model of this study showing the transition of individuals between compartments, derived from assumptions and the formulations of the model.
Linear transition terms are used to describe the movement of individuals between compartments. This choice is consistent with standard practice in compartmental modeling, particularly in epidemiological and socio-dynamic systems, where transitions are assumed to occur at constant per capita rates. For example, the rate at which exposed individuals become active burglars is modeled as proportional to the number of exposed individuals, reflecting a uniform likelihood of transition over time.

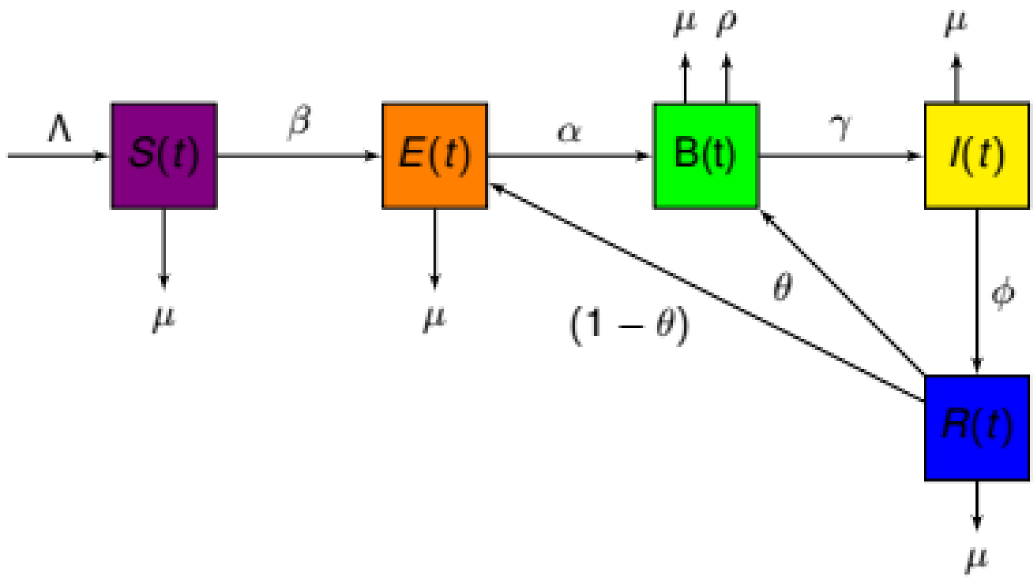

Fig. 2.1. Schematic diagram of the burglary transmission model showing transitions between compartments. Arrows indicate movement between susceptible (S), exposed (E), burglary-active (B), incarcerated (I), and released (R) populations.

2.4 Model equations

From the schematic diagram, the model's system of ordinary differential equations is deduced as shown below;

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - (\beta + \mu)S \\
\frac{dE}{dt} &= \beta S + (1-\theta)R - (\alpha + \mu)E \\
\frac{dB}{dt} &= \alpha E + \theta R - (\rho + \gamma + \mu)B \\
\frac{dI}{dt} &= \gamma B - (\phi + \mu)I \\
\frac{dR}{dt} &= \phi I - (\mu + 1)R
\end{aligned} \tag{2}$$

2.5 Model properties

Analyzing the positivity and boundedness of the model solutions is critical to ensure its mathematical validity and applicability. A model is considered mathematically meaningful if its solutions are both positive and bounded.

2.5.1 Positivity of the solution

Given that the study model describes the human population, all state variables and parameters of equation (1) must remain positive and well defined for all $t > 0$ Let $\Gamma = \{(S, E, B, I, R) \in \mathbb{R}^5_+; S > 0, E > 0, B > 0, I > 0, R > 0\}$. Then the solution of S (t), E (t), B (t), I (t) and R (t) is positive for $t \geq 0$

Proof. Consider

$$\frac{dS}{dt} = \Lambda - (\beta + \mu)S \tag{3}$$
$$\frac{dS}{dt} \geq -(\beta + \mu)$$

By the use of variation of constant formula, that is, integrating both side, the solution of equation 3 is as follows

$$\begin{aligned}
\int \frac{dS}{S} &\geq -\int (\beta + \mu)dt \\
\ln(S)|_0^t &\geq -\int (\beta + \mu)dt \\
\frac{S(t)}{S(0)} &\geq e^{-(\frac{\delta\tau(1-\omega)(B(t)+\lambda E(t)}{N}+\mu)t} \\
S(t) &\geq S(0)e^{-(\frac{\delta\tau(1-\omega)[B(t)+\lambda E(t)}{N}+\mu)t} \geq 0\text{for}\forall_t \geq 0
\end{aligned}$$

This shows that the state variable $S(t)$ is positive for all $t \geq 0$. Similarly, when the same procedure is applied in $E(t)$, $B(t)$, $I(t)$ and $R(t)$, positive solutions obtained were as follows.

$$\begin{aligned}
E(t) &\geq E(0)e^{-(\alpha+\mu)t} \geq 0\text{for}\forall_t \geq 0 \\
B(t) &\geq B(0)e^{-(\rho+\gamma+\mu)t} \geq 0\text{for}\forall_t \geq 0 \\
I(t) &\geq I(0)e^{-(\phi+\mu)t} \geq 0\text{for}\forall_t \geq 0 \\
R(t) &\geq R(0)e^{-(\mu+1)t} \geq 0\text{for}\forall_t \geq 0
\end{aligned} \tag{4}$$

Thus, the proof confirms the non-negativity of the model Equation (1) $\forall_t \geq 0$. □

2.5.2 Boundedness of the Solutions

The total population under the study of model (2) given by $N(t) = S(t) + E(t) + B(t) + I(t) + R(t)$. Differentiating the total population with respect to time (t);

$$\frac{dN(t)}{dt} = \Lambda - \mu N - \rho B$$
$$\frac{dN(t)}{dt} \leq \Lambda - \mu N \tag{5}$$

Using a variation constant formula and integrating both sides, the results obtained are as follows.

$$\int_0^t \frac{dN}{\Lambda - \mu N} \leq \int_0^t dt$$
$$N(t) \leq \frac{\Lambda}{\mu} - \frac{\Lambda - \mu N(0)e^{-\mu t}}{\mu}$$

Let $A = \frac{\Lambda - \mu N(0)}{\mu}$ therefore,

$$N(t) \leq \frac{\Lambda}{\mu} - Ae^{-\mu t} \tag{6}$$

at t = 0, equation 6 become

$$N(0) \leq \frac{\Lambda}{\mu} - Ae^0$$
$$A \geq \frac{\Lambda}{\mu} - N(0)$$

therefore

$$N(t) \leq \frac{\Lambda}{\mu} - (\frac{\Lambda}{\mu} - N_0)e^{-\mu t}$$

by applying initial condition as $t \to \infty$ and the total population $N(t) \to \frac{\Lambda}{\mu}$ then

$$0 \leq N(t) \leq \frac{\Lambda}{\mu}$$

$$N(t) \leq max(\frac{\Lambda}{\mu}, N(0))$$

$$N(t) \leq \frac{\Lambda}{\mu}, \text{otherwise} N(0) \text{is the maximum boundary of N(t)}$$

Therefore, the model studied in the feasible region Γ is obtained as

$$\Gamma = \{S, E, B, I, R\} \in {\mathbb{R}_+}^5; 0 \leq N(t) \leq \frac{\Lambda}{\mu} \quad (7)$$

This shows that the set of solutions is bounded at a maximum $\{\frac{\Lambda}{\mu}\}$. Thus, the model Equation (1) is mathematically well-posed in the region Γ and criminologically meaningful, ensuring that all compartments remain within realistic bounds and reflect key behavioral pathways in burglary dynamics.

3 Model Analysis

3.1 Effective Reproduction number

The effective reproduction number R_e is the average number of people who become burglars as a result of an entry of a burglar into the human population[23]. The effective reproduction number is a very important tool for determining the impact of burglary on the susceptible population. It is used to understand the effect or capacity of burglary in the society when there are no control measures. When there are other measures of reducing burglary, such as creating job opportunities, carrying out intensive patrol by police officers among others, the reproduction number will be maintained less than unity. R_e= Spectral radius of the matrix and is given by;

$$R_e = FV^{-1} \quad (8)$$

Where F is the Jacobian matrix of $\mathcal{F}$ which is the rate of appearance of new burglars in the compartments. V is the Jacobian of $\mathcal{V}$, which is the rate of transfer of individuals in and out of the compartment. It is calculated using the next generation matrix approach[23].
Using the exposed and burglary equations, R_e is computed as follow;

$$\begin{aligned} \frac{dE}{dt} &= \frac{\delta\tau(1-\omega(B(t)+\lambda E(t))}{N(t)}S + (1-\theta)R - (\alpha+\mu)E \\ \frac{dB}{dt} &= \alpha E + \theta R - (\rho+\gamma+\mu)B \end{aligned}$$

We define $\mathcal{F}$ as

$$\mathcal{F} = \begin{pmatrix} \beta S \\ 0 \end{pmatrix}$$

$$\mathcal{V} = \begin{pmatrix} (\alpha+\mu)E - (1-\theta)R \\ (\rho+\gamma+\mu)B - \alpha E - \theta R \end{pmatrix} \tag{9}$$

The Jacobian of $\mathcal{F}$ at the burglary free equilibrium point denoted as F is given by;

$$\mathbb{F} = \begin{pmatrix} \frac{\partial \mathbf{f_1}}{\partial \mathbf{E}} & \frac{\partial \mathbf{f_1}}{\partial \mathbf{B}} \\ \frac{\partial \mathbf{f_2}}{\partial \mathbf{E}} & \frac{\partial \mathbf{f_2}}{\partial \mathbf{B}} \end{pmatrix} = \begin{pmatrix} \frac{\delta\tau(1-\omega)\lambda S}{N} & \frac{\delta\tau(1-\omega)S}{N} \\ 0 & 0 \end{pmatrix} \tag{10}$$

Similarly, the Jacobian of $\mathcal{V}$ at the burglary free equilibrium denoted as V is given by

$$\mathbb{J}_{\mathbb{V}} = \begin{pmatrix} \frac{\partial \mathbf{v_1}}{\partial \mathbf{E}} & \frac{\partial \mathbf{v_1}}{\partial \mathbf{B}} \\ \frac{\partial \mathbf{v_2}}{\partial \mathbf{E}} & \frac{\partial \mathbf{v_2}}{\partial \mathbf{B}} \end{pmatrix} = \begin{pmatrix} \alpha+\mu & 0 \\ -\alpha & \rho+\gamma+\mu \end{pmatrix} \tag{11}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{\alpha+\mu} & 0 \\ \frac{\alpha}{(\rho+\gamma+\mu)(\alpha+\mu)} & \frac{1}{\rho+\gamma+\mu} \end{pmatrix} \tag{12}$$

Therefore,

$$FV^{-1} = \begin{pmatrix} (\frac{\delta\tau(1-\omega)\lambda S}{(\alpha+\mu)N} + \frac{\delta\tau(1-\omega)\alpha S}{N(\alpha+\mu)(\rho+\gamma+\mu)}) & \frac{\delta\tau(1-\omega)S}{N(\rho+\gamma+\mu)} \\ 0 & 0 \end{pmatrix} \tag{13}$$

The largest eigenvalue of FV^{-1} is the effective reproduction number (R_e)

$$\left| \begin{pmatrix} (\frac{\delta\tau(1-\omega)\lambda S}{(\alpha+\mu)N} + \frac{\delta\tau(1-\omega)\alpha S}{N(\alpha+\mu)(\rho+\gamma+\mu)}) - \lambda & \frac{\delta\tau(1-\omega)S}{N(\rho+\gamma+\mu)} \\ 0 & 0-\lambda \end{pmatrix} \right| = 0$$

$$\lambda^2 - \lambda(\frac{\delta\tau(1-\omega)\lambda S}{(\alpha+\mu)N} + \frac{\delta\tau(1-\omega)\alpha S}{N(\alpha+\mu)(\rho+\gamma+\mu)}) = 0$$

$$\lambda^2 - \lambda(\frac{\delta\tau(1-\omega)\lambda S}{(\alpha+\mu)N} + \frac{\delta\tau(1-\omega)\alpha S}{N(\alpha+\mu)(\rho+\gamma+\mu)}) = 0$$

$$\lambda_1 = 0$$
$$\lambda_2 = (\frac{\delta\tau(1-\omega)\lambda S}{(\alpha+\mu)N} + \frac{\delta\tau(1-\omega)\alpha S}{N(\alpha+\mu)(\rho+\gamma+\mu)})$$

In a burglary-free population S=$N = \frac{\Lambda}{\mu}$, Therefore the effective reproduction number of model equation 1 is defined as

$$R_e = \delta\tau(1-\omega)\{\frac{\lambda(\rho+\gamma+\mu)+\alpha}{(\alpha+\mu)(\rho+\gamma+\mu)}\} \tag{14}$$

This determines whether or not burglary will invade the population. Mathematically, it implies that if $R_e < 1$, burglary will die out of the population and when $R_e > 1$, burglary will persist in the population.

3.2 Existence of Burglary Free Equilibrium point(BFE)

The burglary-free equilibrium BFE, denoted by E^o describes a stable population state completely devoid of burglary activity. To obtain a burglary-free equilibrium, we set the normalized model Equation (2) to zero, then consider compartments $E = B = I = R = 0$. Hence,

$$E^0 = \{S(t), E(t), B(t), I(t), R(t)\} = (\frac{\Lambda}{\mu}, 0, 0, 0, 0) \tag{15}$$

3.3 Local Stability of Burglary-free Equilibrium

In this part, we use the Jacobian stability approach to demonstrate the local stability of the burglary free equilibrium points. The burglary free equilibrium E^0 of the model equation (1) is locally asymptotically stable whenever $R_e < 1$.

Proof. Consider the Jacobian matrix of the linearized model equation (1) at BFE given by

$$J_E{}^0 = \begin{pmatrix} -\mu & -\delta\tau(1-\omega)\lambda & -\delta\tau(1-\omega) & 0 & 0 \\ 0 & \delta\tau(1-\omega)\lambda - (\alpha+\mu) & \delta\tau(1-\omega) & 0 & (1-\theta) \\ 0 & \alpha & -(\rho+\gamma+\mu) & 0 & \theta \\ 0 & 0 & \gamma & -(\phi+\mu) & 0 \\ 0 & 0 & 0 & \phi & -(\mu+1) \end{pmatrix} \tag{16}$$

The first eigenvalue $\lambda_1 = -\mu < 0$ by observation, the eigenvalues of the remaining 4×4 matrix is determined by the trace and determinant.

$$J_{BFE} = \begin{pmatrix} \delta\tau(1-\omega)\lambda - (\alpha+\mu) & \delta\tau(1-\omega) & 0 & (1-\theta) \\ \alpha & -(\rho+\gamma+\mu) & 0 & \theta \\ 0 & \gamma & -(\phi+\mu) & 0 \\ 0 & 0 & \phi & -(\mu+1) \end{pmatrix} \tag{17}$$

The trace which is the sum of the main diagonal is given by

$$\varpi(J_E{}^0) = (\delta\tau(1-\omega)\lambda - (\alpha+\mu)) - (\rho+\gamma+\mu) - (\phi+\mu) - (\mu+1) \tag{18}$$

the trace of the above Jacobian matrix remain negative provided $\delta\tau(1-\omega)\lambda \leq (\alpha+\mu)$. The trace in terms of effective reproduction number is give by;

$$\varpi(J_E{}^0) = (R_e - 1)\alpha + \mu - \frac{\delta\tau(1-\omega)\alpha}{(\rho+\gamma+\mu)} - (\rho+\gamma+\mu) - (\phi+\mu) - (\mu+1) \tag{19}$$

The determinant of the above Jacobian matrix is determined by the use of block triangle method where is it partitioned into four sub-matrices as shown below;

$$J_{E^0} = \begin{pmatrix} A & B \\ 0 & D \end{pmatrix},$$

where

$$A = \begin{pmatrix} \delta\tau(1-\omega)\lambda - (\alpha+\mu) & \delta\tau(1-\omega) \\ \alpha & -(\rho+\gamma+\mu) \end{pmatrix}, \quad B = \begin{pmatrix} 0 & (1-\theta) \\ 0 & \theta \end{pmatrix}, \quad D = \begin{pmatrix} -(\phi+\mu) & 0 \\ \phi & -(\mu+1) \end{pmatrix}.$$

By the use the use of block determinant formula,

$$\det(J_{E^0}) = \det(A) \cdot \det(D).$$

$$\det(J_{E^0}) = [(\delta\tau(1-\omega)\lambda - (\alpha+\mu))\,(-(\rho+\gamma+\mu)) - (\delta\tau(1-\omega)\alpha]\,(\phi+\mu)(\mu+1).$$

In terms of R_e it is given by;

$$\det(J_{E^0}) = ((1-R_e)(\alpha+\mu)(\rho+\gamma+\mu)(\phi+\mu)(\mu+1)) \tag{20}$$

The determinant remains positive provided that $R_e < 1$ and all model parameters are positive. This ensures that the model is locally asymptotically stable whenever $R_e < 1$ by the Routh-Hurwitz criterion [3]. □

3.4 Global Stability of the Burglary-free Equilibrium

The global asymptotic stability of the burglary-free equilibrium for model equation 2 can be analyzed through Lyapunov's direct method. The Lyapunov's method is used here to prove global stability, as it uniquely handles the entire nonlinear system while providing convergence guarantees across all feasible initial conditions, a critical requirement for policy applications. BFE is globally asymptotically stable whenever $R_e < 1 \in \Gamma$.

Proof. Consider the Lyapunov function [22] below $P = y_1E + y_2B$ Taking derivative with respect to (t), we have

$$\frac{dP}{dt} = y_1\frac{dE}{dt} + y_2\frac{dB}{dt} \tag{1}$$

By substituting $\frac{dE}{dt}$ and $\frac{dB}{dt}$ from model 1, we get

$$\frac{dP}{dt} = y_1\frac{\delta\tau(1-\omega)B(t)S}{N} + y_1\frac{\delta\tau(1-\omega)\lambda E(t)S}{N} + y_1(1-\theta)R - y_1(\alpha+\mu)E + y_2\alpha E + y_2\theta R - y_2(\rho+\gamma+\mu)B \tag{22}$$

Taking $y_2 = -y_1(\frac{\delta\tau(1-\omega)\lambda-(\alpha+\mu)}{\alpha})$ and $y_1 = 1$

$$\frac{dP}{dt} \le y_1\delta\tau(1-\omega)B(t) - y_2(\rho+\gamma+\mu)B(t) \tag{23}$$

$$\frac{dP}{dt} \le \frac{(\rho+\gamma+\mu)(\alpha+\mu)}{\alpha}(\frac{\delta\tau(1-\omega)\alpha}{\rho+\gamma+\mu)(\alpha+\mu)} + \frac{\delta\tau(1-\omega)\lambda}{\alpha+\mu} - 1)B(t)$$

$$\frac{dP}{dt} \le \frac{(\rho+\gamma+\mu)(\alpha+\mu)}{\alpha}((R_e-1))B(t) \tag{24}$$

□

Hence, from the proof it is clear that $\frac{dP}{dt} \le 0$ if $R_e < 1$ and $\frac{dP}{dt} = 0$ if $B = 0$. Therefore, the solution of Equation (2) satisfies $B \longrightarrow 0$ as $t \longrightarrow \infty$ by LaSalle's principle [22] thus the Burglary free equilibrium point is globally asymptotically stable whenever $R_e < 1$. The criminological implication of this result shows that any small influx of burglars in the population will not increase the number of burglars, thus the burglary will die out in the population.

3.5 Burglary Endemic Equilibrium

The endemic equilibrium point $E^* = (S^*, E^*, B^*, I^*, R^*)$ represents a steady-state solution of the burglary transmission model where criminal activity persists in the population. It is computed by setting model (2) to zero. By the use of back substitution, it is given by;

$$
\begin{aligned}
S^* &= \frac{x}{y} \\
E^* &= \frac{((\rho+\gamma+\mu)(\mu+1)(\phi+\mu) - \theta\gamma\phi)(N\alpha\gamma\phi((1-\theta)+\mu)(\phi+\mu)(\Lambda y - \mu x))}{\alpha(\phi+\mu)(\mu+1)xy} \\
B^* &= \frac{N\alpha((\phi+\mu)(\mu+1)(\Lambda y - \mu x))}{xy} \\
I^* &= \frac{N\alpha\gamma((\phi+\mu)(\mu+1)(\Lambda y - \mu x))}{(\phi+\mu)xy} \\
R^* &= \frac{N\alpha\gamma\phi((\phi+\mu)((1-\theta)+\mu)(\Lambda y - \mu x))}{(\phi+\mu)(\mu+1)xy}
\end{aligned} \tag{25}
$$

where x and y are given by

$$
\begin{aligned}
x &= N((\alpha+\mu)(\rho+\gamma+\mu)(\mu+1)(\phi+\mu) - (\alpha+\mu)\theta\gamma\phi - (1-\theta)\alpha\gamma\phi) \\
y &= R_e(\alpha+\mu)(\rho+\gamma+\mu)(\mu+1)(\phi+\mu) - \delta\tau(1-\omega)\lambda\theta\gamma\phi
\end{aligned}
$$

3.6 Local Stability of Burglary Endemic Equilibrium

For burglary to be endemic in a susceptible population $E^* > 0$, whenever $R_e > 1$. At this point, the persistence of burglary occurs in the population thus $B(t) > 0$. Consider the Jacobian matrix of the linearized model equation (2), evaluated at the endemic equilibrium point, as given by the following.

$$
J_{E}{}^* = \begin{pmatrix} -T_1 - \mu & -T_3 & -T_3 & 0 & 0 \\ T_1 & T_2 & T_3 & 0 & (1-\theta) \\ 0 & \alpha & -(\rho+\gamma+\mu) & 0 & \theta \\ 0 & 0 & \gamma & -(\phi+\mu) & 0 \\ 0 & 0 & 0 & \phi & -(\mu+1) \end{pmatrix} \tag{26}
$$

Where $T_1 = \frac{(\delta\tau(1-\omega)(B^*+\lambda E^*))}{N}$, $T_2 = \frac{\delta\tau(1-\omega)\lambda S^*}{N} - (\alpha+\mu)$ and $T_3 = \frac{\delta\tau(1-\omega)\lambda S^*}{N}$
By the Routh-Hurwitz criterion, the eigenvalues of the above Jacobian matrix are determined by computing the trace and determinant [13]. For the trace to be negative, this condition must hold, $\frac{\delta\tau(1-\omega)\lambda S^*}{N} < (\alpha+\mu)$
The determinant is given by

$$
DetJ_{E^*} = T_2(T_1+\mu)(\rho+\gamma+\mu)(\phi+\mu)(\mu+1) + \alpha T_3(T_1+\mu)(\phi+\mu)(\mu+1) - T_1T_3(\phi+\mu)(\mu+1)((\rho+\gamma+\mu)+\alpha) - T_2(T_1+\mu \tag{27}
$$

The determinant remains positive provided that all the model parameters are positive and $R_e > 1$. Thus, E^* is locally asymptotically stable whenever $R_e > 1$ by the Routh-Hurwitz. The criminological implication of these results is that a small influx of active burglars into the susceptible population will lead to an outbreak of burglary in society.

3.7 Global Stability of Burglary Endemic Equilibrium

To rigorously establish the global stability of the endemic equilibrium E^*, we employ LaSalle invariance principle[7] and Lyapunov's direct method. This approach provides conclusive stability results across the entire feasible domain Γ without restricting analysis to small perturbations of the equilibrium. Consider a non-linear Lyapunov function. $U : (S, E, B, I, R) \in \Gamma \subset \mathbb{R}^5_+ : S, E, B, I, R > 0$ defined as

$$
U = m_1(S - S^* - S^* \ln\frac{S}{S^*}) + m_2(E - E^* - E^* \ln\frac{E}{E^*}) + m_3(B - B^* - B^* \ln\frac{B}{B^*}) + m_4(I - I^* - I^* \ln\frac{I}{I^*}) + m_5(R - R^* - R^* \ln\frac{R}{R^*}) \tag{28}
$$

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Where U is in the interior region Γ. E^* is the global minimum of U in Γ and $U : \{S(t), E(t), B(t), I(t), R(t)\} = 0$. The time derivative of U along the solution of model equation (28) is given by; $U^1 = m_1(1 - \frac{S^*}{S})S' + m_2(1 - \frac{E^*}{E})E' + m_3(1 - \frac{B^*}{B})B' + m_4(1 - \frac{I^*}{I})I' + m_5(1 - \frac{R^*}{R})R'$ At endemic taking E^* into account and $m_i > 0$ for $i = 1, 2, 3, 4, 5$ in Γ and its constants m_i are continuously differentiable in Γ and $U(\Gamma^*) = 0$, where $\Gamma = (S^*, E^*, B^*, I^*, R^*)$. Global stability of endemic hold if $U' \leq 0$. Therefore, at endemic equilibrium U' is given as follows;

$$\begin{aligned}U' = m_1(1 - \frac{S^*}{S})\,((\beta + \mu)S^* - (\beta + \mu)S) + m_2(1 - \frac{E^*}{E})((\alpha + \mu)E^* - (\alpha + \mu)E) + m_3(1 - \frac{B^*}{B})((\rho + \gamma + \mu)B^* - (\rho + \gamma + \mu)B) \\ + m_4(1 - \frac{I^*}{I})((\phi + \mu)I^* - (\phi + \mu)I) + m_5(1 - \frac{R^*}{R})(\mu + 1)R^* - ((1 - \theta) + \mu)R)\end{aligned} \tag{29}$$

The factorization of terms in Equation (29) leads to;

$$\begin{aligned}U' = m_1(\frac{S - S^*}{S})(-(\beta + \mu)S(1 - \frac{S^*}{S})) + m_2(\frac{E - E^*}{E})(-(\alpha + \mu)E(1 - \frac{E^*}{E})) + m_3(\frac{B - B^*}{B})(-(\rho + \gamma + \mu)B)(1 - \frac{B^*}{B})) \\ + m_4(\frac{I - I^*}{I})(-(\phi + \mu)I(1 - \frac{I^*}{I})) + m_5(\frac{R - R^*}{R})(-(\mu + 1)R(1 - \frac{R^*}{R}))\end{aligned} \tag{30}$$

By opening the bracket

$$U' = -m_1(\beta + \mu)S(\frac{S - S^*}{S})^2 - m_2(\alpha + \mu)E(\frac{E - E^*}{E})^2 - m_3(\rho + \gamma + \mu)B(\frac{B - B^*}{B})^2 - m_4\phi + \mu)I(\frac{I - I^*}{I})^2 - m_5(\mu + 1)R(\frac{R -}{R}$$

From the previous analysis, we observed that all parameters are positive and all state variables $S, E, B, I, R \geq 0$. We also note that $(\frac{S-S^*}{S})^2 \geq 0, (\frac{E-E^*}{E})^2 \geq 0, \frac{B-B^*}{B})^2 \geq 0, (\frac{I-I^*}{I})^2 \geq 0, (\frac{R-R^*}{R})^2 \geq 0$

$$U' = -m_1(\beta + \mu)S(\frac{S - S^*}{S})^2 - m_2(\alpha + \mu)E(\frac{E - E^*}{E})^2 - m_3(\rho + \gamma + \mu)B(\frac{B - B^*}{B})^2 - m_4\phi + \mu)I(\frac{I - I^*}{I})^2 - m_5(\mu + 1)R(\frac{R -}{R} \tag{31}$$

Hence, $U' \leq 0$ and $U' = 0$ when $S = S^*, E = E^*, B = B^*, I = I^*,\ R = R^*$. Thus, the largest compact invariant set in $S(t), E(t), B(t), I(t), R(t) \in \Gamma$: $U = 0$ is a singleton E^*, where E^* is the endemic equilibrium point. Therefore,E^* is globally asymptotically stable in the interior of Γ. This means that any perturbation of the system caused by the introduction of additional burglars in the susceptible population will not destabilize the equilibrium and the system will return to the endemic state E^*[12].

3.8 Sensitivity Analysis

Sensitivity analysis is the process of investigating the parameters that have a greater impact on the effective reproduction number R_e. This process helps to know the criteria to curb the burglary problem in susceptible population as directed by that parameter which has a greater impact. We use the normalized forward sensitivity index to know the elasticity[11]. The normalized forward sensitivity index of the reproduction number R_e in Equation (14) with respect to all parameters is shown below. The normalized forward sensitivity index in given by;

$$Y_x^{R_e} = \frac{x}{R_e}.\frac{\partial R_e}{\partial x} \tag{32}$$

where x represent the basic parameters that define effective reproduction number. Using the above the elasticity $Y_x^{R_e}$ the results obtained are as follows;

$$\begin{aligned}
Y_\delta^{R_e} &= \frac{\delta}{R_e}.\frac{\partial R_e}{\partial \delta} = 1\\
Y_\tau^{R_e} &= \frac{\tau}{R_e}.\frac{\partial R_e}{\partial \tau} = 1\\
Y_\omega^{R_e} &= (\frac{-\omega}{1-\omega})\\
Y_\lambda^{R_e} &= \frac{(\rho+\gamma+\mu)}{(\rho+\gamma+\mu)+\alpha}\\
Y_\rho^{R_e} &= (\frac{\rho}{(\rho+\gamma+\mu)+\alpha} - \frac{\rho}{(\rho+\gamma+\mu)})\\
Y_\gamma^{R_e} &= (\frac{\gamma}{(\rho+\gamma+\mu)+\alpha} - \frac{\gamma}{(\rho+\gamma+\mu)})\\
Y_\mu^{R_e} &= (\frac{\mu}{(\rho+\gamma+\mu)+\alpha} - \frac{2\mu^2+\mu(\rho+\gamma+\alpha)}{(\alpha+\mu)(\rho+\gamma+\mu)})\\
Y_\alpha^{R_e} &= (\frac{\alpha}{\lambda(\rho+\gamma+\mu)+\alpha} - \frac{\alpha}{(\alpha+\mu)})
\end{aligned} \tag{33}$$

Table 2. Sensitivity indices of R_e with respect to the model parameters

Parameters	Description	Values	Sensitivity Index	Sources
δ	Rate at which individual become burglar upon contact	0.95week^{-1}	1	[11]
τ	contact rate	0.95 week^{-1}	1	Assumed
λ	Exposure influence rate	0.4week^{-1}	0.7508	
ω	Employment rate	0.4week^{-1}	-0.6667	Assumed
α	Rate of transfer from exposed to burglary	0.15week^{-1}	-.4806	[6]
μ	Natural death rate	0.05	-0.2776	[11]
γ	Incarceration rate	0.4week^{-1}	-0.2205	[16]
ρ	Burglary specific death rate	0.002week^{-1}	-0.0011	[4]

From Table 2, we can conclude that some sensitivity values of the parameters that define R_e are positive while other are negative. Analytically, it indicates that all values of R_e for the sensitivity index that are negative are the most important factors to be considered in controlling burglary in a population. Therefore, increasing the value of such parameters become the most control criteria of burglary. The employment rate from the table is the most negative sensitivity parameter to be considered. This shows that a decrease in the value of ω will subsequently increase the number of unemployed individuals, thus increasing burglary practices in Kenya. Therefore, the Kenyan government and society in general should come up with suitable strategies on how to increase ω to alleviate the prevalence of burglary in the community.

4 Numerical simulation

Numerical simulations were performed to explore the dynamics of burglary under varying employment rates, using parameter values derived from Kenyan data sources such as the Kenya National Bureau of Statistics (KNBS, 2023), National Crime Research Centre (NCRC, 2021), and Kenya Prison Service (2022 Annual Report). For parameters lacking empirical estimates, reasonable assumptions were made based on prior literature.Initial conditions; $(S_0, E_0, B_0, I_0, R_0) = (1000, 700, 200, 160, 45)$

Table 3. Parameter values used in simulations

Parameters	description	values	sources
$S(t)$	Susceptible class	1000	Estimate
$E(t)$	Exposed class	700	Estimate
$B(t)$	Burglar class	200	Estimate
$I(t)$	Incarcerated class	160	Estimate
$R(t)$	Released class	45	Estimate
Λ	Recruitment rate	50	[24]
θ	Recidivism rate	0.9	[15]
$(1-\theta)$	Rate at which released relapse to exposed class	0.75	[15]
δ	Rate at which individual become burglar upon contact	0.95	[11]
τ	Contact rate	0.95	Assumed
λ	Exposure influence rate	0.9	Assumed
α	Rate of transfer from exposed to burglary	0.15	[6]
ω	Employment rate	0.4	[5]
ρ	Burglary associated death rate	0.002	[4]
μ	Natural death rate	0.05	[11]
γ	Incarceration rate	0.4	[16]
ϕ	Removal rate from prion	0.5	[18]

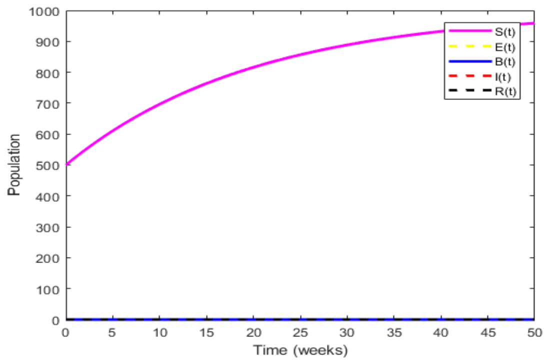

Fig. 4.1. Simulation results showing the population when $R_e = 0.5287$ with other parameters made constant.

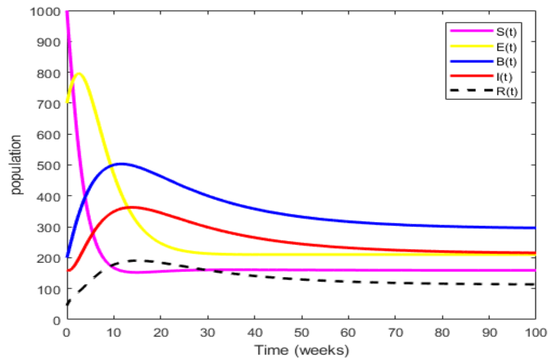

Fig. 4.2. Simulation showing burglary dynamics in the population when $R_e = 3.1724$ with other parameters made constant

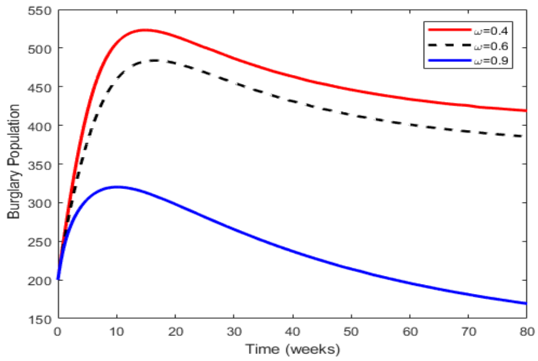

Fig. 4.3. Burglary prevalence under the variation of employment rate (ω)

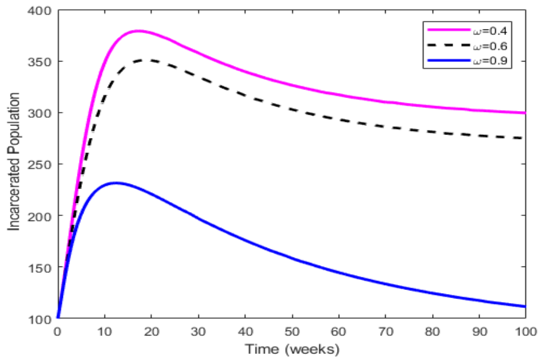

Fig. 4.4. Dynamics of Incarcerated population with the variation of employment rate (ω)

When the effective reproduction number $R_e = 0.5287$ (< 1), achieved with an employment rate $\omega = 0.9$, the susceptible population $S(t)$ grew exponentially before stabilizing at $\frac{\Lambda}{\mu}$ (1000 individuals), since recruitment balanced natural mortality. The exposed $E(t)$, burglary $B(t)$, incarcerated $I(t)$, and released $R(t)$ populations remained negligible, converging to zero over time. This demonstrates that high employment rates can effectively suppress burglary to negligible levels, highlighting the importance of job creation initiatives for crime reduction.

In contrast to the graph in Figure 2, when $R_e = 3.1724$ (> 1) with $\omega = 0.4$ unemployment rate $= 0.6$, the susceptible population declined sharply due to high transmission rates β, stabilizing at approximately 400 individuals. The exposed class peaked at $t = 4$ weeks before declining as individuals transitioned to the burglary class or exited the system. The burglary population reached its maximum at $t = 12$ weeks, then decreased due to incarceration but stabilized at around 200 individuals, indicating persistent burglary activity. The incarcerated and released classes followed similar trends, with recidivism ($\theta = 0.9$) contributing to re-exposure and sustaining the burglary-endemic state. These results underscore how low employment rates perpetuate burglary by increasing the force of influence β and transmission rates.

Figure 4 demonstrates the inverse relationship between employment rates and burglary prevalence. At the baseline employment rate ($\omega = 0.4$, equivalent to 60% unemployment), burglary activity reaches its peak level of approximately 200 individuals. As employment opportunities increase to $\omega = 0.6$ and $\omega = 0.9$, we observe a progressive 62.5% and 75% reduction in burglary cases respectively. This nonlinear response aligns with findings by [13], confirming that economic deprivation significantly elevates property crime rates. The results quantitatively support [21]'s economic theory of crime, where reduced legitimate earning opportunities ($1 - \omega > 0.5$) increases the relative attractiveness of illegal income sources. The transition dynamics show that improving employment from $\omega = 0.4$ to $\omega = 0.9$ produces disproportionately large reductions in burglary, suggesting the existence of critical economic thresholds for crime prevention.

Figure 5 reveals the downstream effects of employment rates on incarceration dynamics. With low employment ($\omega = 0.4$), the incarcerated population $I(t)$ peaks at 120 individuals by week 12 before stabilizing at approximately 80. This reflects both the high burglary recruitment rate and the system's limited capacity for rehabilitation. At moderate employment ($\omega = 0.6$), we observe a 33% reduction in peak incarceration (80 individuals) occurring 4 weeks later. Most significantly, at $\omega = 0.9$, the system maintains incarceration below 45 individuals throughout the simulation period, with equilibrium levels 43.75% lower than the baseline scenario. The temporal shift in peak incarceration (from week 12 to week 20) suggests that employment interventions may require 8-10 weeks to measurably reduce pressure on the criminal justice system. These findings collectively demonstrate that employment creation serves as both:

- A preventive measure that is reducing burglary incidence through $\omega \uparrow \rightarrow R_e \downarrow$
- A corrective mechanism that is lowering recidivism through $\omega \uparrow \rightarrow \theta \downarrow$

The 2.5 fold difference in equilibrium incarceration between $\omega = 0.4$ and $\omega = 0.9$ scenarios underscores the potential for labor market interventions to reduce long-term prison system burdens. However, the delayed response (8-10 week lag) suggests that employment programs should be implemented proactively rather than as crisis responses.

5 Discussion and Conclusion

This study developed a mathematical model to analyze how unemployment influences burglary dynamics in Kenya. Our analysis reveals that the effective reproduction number (R_e) is most sensitive to the employment rate (ω), with a 10% increase in employment reducing R_e. When employment rates fall below 40% ($\omega = 0.4$), the model predicts endemic burglary ($R_e = 3.1724$), while increasing employment to 90% ($\omega = 0.9$) drives R_e below 1 ($R_e = 0.5287$), creating conditions for burglary eradication. These findings quantitatively demonstrate that employment creation is the most effective single intervention to reduce burglary, outperforming even higher incarceration rates in our sensitivity analysis ($Y_\omega^{R_e} = -0.6667$ vs $Y_\gamma^{R_e} = -0.2205$).

The model reveals two equilibria; a stable burglary-free state ($R_e < 1$) and a persistent endemic state ($R_e > 1$). The burglary-free equilibrium ($R_e < 1$) proves globally stable when maintained, suggesting that employment programs can create lasting reductions in burglary rates. The endemic equilibrium ($R_e > 1$) emerges persistently when unemployment exceeds 60%, with our simulations showing that this leads to 2.5 times higher incarceration rates compared to high-employment scenarios. In particular, we observed an 8-week lag between employment increases and measurable reductions in incarceration, indicating that job creation initiatives require sustained implementation before yielding full benefits.

Our results highlight three key mechanisms through which employment reduces burglary that is; by decreasing the force of infection (β) through reduced susceptible-burglar interactions, by lowering recidivism rates among released offenders (θ), and by creating alternative income sources that reduce the relative attractiveness of burglary. The nonlinear response curve suggests a particular value in increasing employment from 40% to 70%, where we observe the steepest reductions in burglary practices.

Although the model provides valuable insights, we acknowledge limitations including its homogeneous mixing assumption and static parameter values. Nevertheless, employment creation emerges as the most effective intervention, reducing burglary more significantly than punitive measures. The quantified relationship between ω and R_e offers policymakers specific targets for economic interventions, suggesting that employment programs should aim for at least 70% coverage to achieve a meaningful reduction in crime.

6 Recommendations and Future work

Burglary remains a significant socioeconomic challenge in Kenya, imposing substantial costs on law enforcement, judicial systems, and correctional facilities. Our model demonstrates that employment creation programs ($\omega \geq 0.7$) could reduce burglary incidence by up to 75% while decreasing incarceration rates by 60%, offering a cost-effective alternative to purely punitive approaches. We recommend immediate implementation of targeted job creation initiatives in high-risk areas, coupled with vocational training programs to address the specific unemployment-to-crime transmission pathways identified in our analysis (β reduction of 0.95 to 0.35 at $\omega = 0.8$).

For law enforcement agencies, our findings suggest optimal intervention timing occurs 8-10 weeks after employment program initiation, when the model predicts maximum synergy between economic and policing strategies. This phased approach could reduce recidivism rates (θ) from 0.9 to 0.6 while maintaining public safety. Policymakers should prioritize evidence-based allocation of resources, focusing on parameters with highest sensitivity indices ($Y_\omega^{R_e} = -0.6667$, $Y_\gamma^{R_e} = -0.2205$). For future work, other researchers should check the following:

1. This study can be extended to consider incorporating other intervention such as vocational training control strategies, government inclusivity, Guidance and canceling criterion.
2. Optimal control analysis can also be considered for designing cost-effective interventions that achieve the desired impact at the minimal cost.

References

[1] Aburili, R. M. (2017). Access to criminal justice in Kenya: An assessment of legal, policy and institutional frameworks. *Doctoral dissertation, University of Nairobi.*

[2] Becker, G. S. (1968). Crime and punishment: An economic approach. *Journal of Political Economy*, 76(2), 169–217.

[3] De Leon, C. V. (2009). Construction of Lyapunov functions for classic SIR, SIS, and SIRS endemic models with variable population size. *Unite Académica de Matemáticas, Facultad de Estudios Superiores Zaragoza, UNAM, Mexico.*

[4] Independent Policing Oversight Authority (IPOA). (2022). *Annual report on police conduct and accountability.* https://www.ipoa.go.ke

[5] Kenya National Bureau of Statistics (KNBS). (2023). *Economic survey 2023.* https://www.knbs.or.ke

[6] Kenya National Bureau of Statistics (KNBS). (2022). *Kenya crime and safety survey report.* https://www.knbs.or.ke

[7] La Salle, J. P. (1976). *The stability of dynamical systems.* Society for Industrial and Applied Mathematics.

[8] Mataru, B., Abonyo, O. J., & Malonza, D. (2023). Mathematical model for crimes in developing countries with some control strategies. *Journal of Applied Mathematics*, 2023, 1–12.

[9] Mbogo, A. M., & Wambua, P. (2022). Assessing the sociopolitical security determinants of crime incidents in Kenya: A case of Kibera informal settlement. *African Journal of Social Sciences*, 12(3), 45–60.

[10] Mburu, L. W., & Bakillah, M. (2016). Modeling spatial interactions between areas to assess the burglary risk. *ISPRS International Journal of Geo-Information*, 5(4), 47.

[11] Mikucki, M. A. (2012). Sensitivity analysis of the basic reproduction number for infectious disease models. *Master's thesis, Colorado State University.*

[12] Misra, A. K., & Singh, A. K. (2011). A mathematical model for unemployment. *Nonlinear Analysis: Real World Applications*, 12(1), 128–136.

[13] Munoli, S. B., & Gani, S. (2016). Optimal control analysis of a mathematical model for unemployment. *Optimal Control Applications and Methods*, 37(4), 798–806.

[14] Musoi, K. (2014). Crime dynamics in urban slums: Evidence from Kibra, Bondeni, Manyatta, and Mishomoroni. *Kenya Policy Briefs*, 8(1), 1–15.

[15] National Crime Research Centre (NCRC). (2021). *Annual crime and safety survey report.* https://www.crimeresearch.go.ke

[16] National Police Service (NPS). (2022). *Crime statistics report.* https://www.nationalpolice.go.ke

[17] Okuom, J., Obange, N., & Odhiambo, S. (2023). Effect of per capita income on youth unemployment in Kenya. *International Journal of Economics*, 8(2), 1–18.

[18] Kenya Prison Service. (2022). *Annual report 2022.* https://www.prisons.go.ke

[19] Siegel, D., Bunt, H., & Zaitch, D. (Eds.). (2003). *Global organized crime: Trends and developments*. Springer.

[20] Soemarsono, A. R., Fitria, I., Nugraheni, K., & Hanifa, N. (2021). Analysis of mathematical model on impact of unemployment growth to crime rates. *Journal of Physics: Conference Series*, 1726(1), 012003.

[21] Sundar, S., Tripathi, A., & Naresh, R. (2018). Does unemployment induce crime in society? A mathematical study. *American Journal of Applied Mathematics and Statistics*, 6(2), 44–53.

[22] Temesgen, D. K., Makinde, O. D., & Legesse, O. L. (2021). Impact of temperature variability on SIRS malaria model. *Journal of Biological Systems*, 29(3), 773–798.

[23] Van den Driessche, P., & Watmough, J. (2008). Further notes on the basic reproduction number. In F. Brauer, P. van den Driessche, & J. Wu (Eds.), *Mathematical epidemiology* (pp. 159–178). Springer.

[24] World Bank. (2023). *Kenya economic update: Inequality and growth*. https://www.worldbank.org/en/country/kenya